

$$\cot^4 y + \csc^2 y = \csc^4 y - \cot^2 y$$

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$$= (\csc^2 y - 1)^2 + \csc^2 y$$


$$= \csc^4 y - 2 \csc^2 y + 1 + \csc^2 y$$

$$= \csc^4 y - \csc^2 y + 1$$

$$= \csc^4 y - (\csc^2 y - 1)$$

$$= \csc^4 y - \cot^2 y \quad \boxed{\text{QED}}$$

$$\frac{1 + \cos \theta}{\tan \theta + \sin \theta} = \cot \theta$$


$$= \frac{1 + \cos \theta}{\frac{\sin \theta}{\cos \theta} + \sin \theta} \cdot \frac{\cos \theta}{\cos \theta}$$

$$= \frac{\cos \theta (1 + \cos \theta)}{\sin \theta + \sin \theta \cos \theta}$$

$$= \frac{\cos \theta (1 + \cancel{\cos \theta})}{\sin \theta (1 + \cancel{\cos \theta})}$$

$$= \frac{\cos \theta}{\sin \theta}$$

$$= \cot \theta$$

QED

$$\frac{\cos(-\beta) - \sec \beta}{\csc \beta + \sin(-\beta)}$$

$$= \frac{\cos \beta - \sec \beta}{\csc \beta - \sin \beta}$$

$$= \frac{\cos \beta - \frac{1}{\cos \beta}}{\frac{1}{\sin \beta} - \sin \beta}$$

$$= \frac{\frac{\cos^2 \beta - 1}{\cos \beta}}{\frac{1 - \sin^2 \beta}{\sin \beta}}$$

$$= \frac{-\frac{\sin^2 \beta}{\cos \beta}}{\frac{\cos^2 \beta}{\sin \beta}}$$

$$= -\frac{\sin^2 \beta}{\cos \beta} \cdot \frac{\sin \beta}{\cos^2 \beta}$$

$$= -\frac{\sin^3 \beta}{\cos^3 \beta}$$

$$= -\tan^3 \beta$$

$$\frac{\tan^2 t - \sec^2 t}{1 - \cos^2 t}$$

$$= \frac{-1}{\sin^2 t}$$

$$= -\csc^2 t$$

Prove $\frac{\cot x - \tan x}{\cos x + \sin x} = \frac{\cos x - \sin x}{\cos x \sin x}$.

$$= \frac{\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}}{\cos x + \sin x} \cdot \frac{\cos x \sin x}{\cos x \sin x}$$

$$= \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x) \cos x \sin x}$$

$$= \frac{(\cancel{\cos x + \sin x})(\cos x - \sin x)}{(\cancel{\cos x + \sin x}) \cos x \sin x}$$

$$= \frac{\cos x - \sin x}{\cos x \sin x}$$

QED